

Remarks on the methods of investigations of alignment of galaxies

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Received _____; accepted _____

ABSTRACT

In the 1975 Hawley and Peebles gave the proposal to use three statistical tests for investigations of the galaxies orientation in the large structures. Nowadays, it has been considered as the standard method of searching for galactic alignments. In the present paper we analyzed the tests in details and proposed a few improvements. Basing on the improvements, the new method of analysis of the alignment of galaxies in clusters is proposed. The power of this method is demonstrated on the sample of 247 Abell clusters with at least 100 objects in each. The distributions of the position angles for galaxies in each cluster are analyzed using statistical tests: χ^2 , Fourier, autocorrelation and Kolmogorow test. The mean value of analyzed statistics is compared with theoretical predictions as well as with results obtained from numerical simulations. We performed 1000 simulations of 247 fictious clusters, each with numbers of galaxies the same as the real ones. We found that orientations of galaxies in analyzed clusters are not random i.e. that there exists an alignment of galaxies in rich Abell galaxy clusters.

Subject headings: galaxies: clusters: general

1. Introduction

The analysis of the orientation of galaxies' planes is regarded as a standard test of galaxies formation scenarios (Peebles 1969; Zeldovich 1970; Sunyaew & Zeldovich 1972; Doroshkevich 1973; Shandarin 1974; Dekel 1985; Wesson 1982; Silk 1983; Bower et al. 2006). Studies of the galaxies' planes orientation were conducted as early as in 19th century (Abbe 1875). The review of early methods and results of investigation performed up to the Second World War can be found in the article of Danver (1942), today only of historical value. The first postwar work, which has been cited to this day, is the treatise of Holmberg (1946), who compared the numbers of galaxies seen face-on and edge-on, discussed the observational effects related to optical measurements of size of galaxy axes, and proved that the observed excess of edge-on galaxies is just of observational origin.

In the early period after the Second World War the researchers were usually investigating distributions of position angles within the galaxy-rich regions (Cetus, Pisces, Hydra, Sextant, Ursa Major, Virgo and Eridanus) of the sky (Wyatt & Brown 1955; Brown 1964, 1968). In his two papers Brown (Brown 1964, 1968) discovered a departure from isotropy in the distributions of position angles. By analyzing the distributions of position angles of large semiaxes of galaxies, Reinhard (1970) and Reinhard & Roberts (1972) and later Nilson (1974) found a very weak preference of galaxy plane alignment with the equator plane of the Local Supergalaxy. However these results are undermined by the presence of background objects.

Further important progress in the investigation of galaxies planes orientation was made by Hawley & Peebles (1975). They discussed in a detailed manner the method of investigating the galaxies' orientation through analyzing distribution of position angles as well as the influence of possible errors and observational effects. In particular, on the earlier papers of Brown, they indicated their insufficient certainty of his results due to possible

errors in observations.

Hawley & Peebles (1975) analyzed the distributions of position angles using χ^2 test, Fourier tests and autocorrelation test. Since Hawley & Peebles (1975) this method was accepted as standard method for analysis of an galactic alignment (Thompson 1976; Mac Gillivray et al. 1982; Djorgovski 1983; Flin & Godłowski 1986; Kindl 1987; Flin 1988; Flin & Godłowski 1989, 1990; van Kampen & Rhee 1990; Cerne & Peterson 1990; Godłowski 1993, 1994; Hu et al. 1995; Godłowski, Baier & MacGillivray 1998; Wu et al. 1997; Godłowski & Ostrowski 1999; Aryal & Saurer 2000; Baier, Godłowski & MacGillivray 2003; Aryal & Saurer 2004; Godłowski et al. 2005; Aryal & Saurer 2005a,b,c, 2006a; Aryal, Kandel & Saurer 2006b; Hu et al 2006; Wu et al. 1997; Wu 2006; Aryal, Paudel & Saurer 2007, 2008; Godłowski & Flin 2010; Godłowski et al. 2010; Aryal, Bachchan & Saurer 2010; Aryal 2011) One should note that there are several modifications and improvements of original Hawley & Peebles (1975) methods (Flin & Godłowski 1986; Kindl 1987; Godłowski 1993, 1994; Godłowski, Baier & MacGillivray 1998; Aryal & Saurer 2000; Godłowski et al. 2010). The aim of the present paper is to present deeper improvements of the original Hawley & Peebles (1975) method and show their usefulness for analysis of galactic orientations in clusters. The power of this method is shown on the sample of 247 rich Abell clusters.

Following Godłowski et al. (2005) suggestion that alignment should increase with richness of the cluster, it was found in Godłowski et al. (2010) that in rich Abell clusters the non-randomness of the galaxies' orientation increased with number of objects in clusters. The question which arose, is if we could say that in analyzed sample of 247 Abell clusters with at least 100 objects each, we found an alignment. For this reason, in the present paper, we analyze the distributions of position angles of galaxies belonging to investigated clusters using χ^2 test, Fourier tests and autocorrelation test applied by Hawley & Peebles (1975)

(see also Flin & Godłowski (1986); Godłowski (1993, 1994); Godłowski & Flin (2010); Godłowski et al. (2010)) as well as Kolmogorow test. For our sample of 247 Abell clusters, we compute the mean values of analyzed statistics. Our null hypothesis H_0 is that the mean value of the analyzed statistics is as expected in the cases of a random distribution of analyzed angles. We compared our results with theoretical predictions as well as with results obtained from numerical simulations.

2. Observational data

Our observational basis is the same as in Godłowski et al. (2010). It is the sample of 247 Abell clusters with at least 100 objects each, taken from PF catalogue (Panko & Flin 2006). The structures were extracted from the Muenster Red Sky Survey (MRSS hereafter) (Ungruhe et al. 2003). MRSS is an optical large scale survey covering the area of 5000 square degrees in the southern hemisphere with $b < -45^\circ$. After scanning 217 ESO plates, it gives the information about 5,5 million galaxies. The 2D Voronoi tessellation technique was applied to the MRSS galaxy catalogue to search for overdense regions (Panko et al. 2009). PF catalogue, like MRSS, is statistically complete till magnitude value $m = 18^m.3$ and it contains structures having at least ten members between magnitude range m_3 and $m_3 + 3$ in each structure field. The m_3 is the magnitude of the third brightest galaxy located in the considered structure region. The resulting PF includes 6188 such structures. We select sample of rich clusters (at least 100 members) being identified with one of ACO clusters (Abell et al. 1989). There are 239 such objects in the PF catalogue. Moreover, we include 9 objects which can be identified with two ACO clusters, which increase our sample to 248 objects. However, we exclude from our analysis A3822, which potentially has substructures (Biviano et al. 1997, 2002). Therefore, our sample has 247 objects.

The data for each galaxy member is taken from the MRSS. These includes: the

equatorial coordinates of galaxies(α , δ), the diameters of major and minor axes of the galaxy image (a and b respectively) and the position angle of the major axis, p . Position angles are recomputed from MRSS clockwise system to standard counterclockwise system. We perform our computation both in Equatorial and Supergalactic Coordinate System (Flin & Godłowski 1986). Because position angles for the face-on galaxies give only marginal information connected with orientation of galaxy, we exclude from analysis all galaxies with axial ratio $q = b/a > 0.75$.

3. The method of investigation

The aim of our paper is to check if orientations of galaxies in investigated clusters are isotropic. In order to check it we test if the distribution of galaxy position angles p (or supergalactic position angles P) is isotropic. We apply statistical tests originally introduced by Hawley & Peebles (1975) and later modified by us Godłowski (1993, 1994); Godłowski et al. (2010), as well as Kolmogorow test. In all considered tests, the entire range of the tested θ angle (where for θ one can put p or P respectively) is divided into n bins of equal width. In the present paper we use $n = 36$.

Let N denote the total number of galaxies in the considered cluster, and N_k - the number of galaxies with orientations within the k -th angular bin. Moreover, $N_{0,k}$ denotes expected number of galaxies in the k -th bin. In our case all $N_{0,k}$ are equal N_0 , which is also mean number of galaxies per bin.

Our first test is χ^2 test:

$$\chi^2 = \sum_{k=1}^n \frac{(N_k - N p_k)^2}{N p_k} = \sum_{k=1}^n \frac{(N_k - N_{0,k})^2}{N_{0,k}}. \quad (1)$$

where p_k is a probability that chosen galaxy falls into k th bin. We divided entire range of a θ angle into n bins, which gives in the χ^2 test $(n - 1)$ degrees of freedom. It means

that expected value $E(\chi^2) = n - 1$ while variance $\sigma^2(\chi^2) = 2(n - 1)$. For $n = 36$ it gives $E(\chi^2) = 35$ and variance $\sigma^2(\chi^2) = 70$.

We analyzed sample of $m = 247$ clusters. So we compute the mean value of analyzed statistics i.e. in discussed case χ^2 value, for whole sample of clusters. If we assume uniform distribution of a θ angle, than expected value $E(\bar{\chi}^2)$ is again equal 35 while variance $\sigma^2(\bar{\chi}^2) = \frac{\sigma^2(\chi^2)}{m} = 0.2834$. It gives in our case standard deviation $\sigma(\bar{\chi}^2) = 0.5324$. We check our theoretical prediction by numerical simulations. We do this in two ways.

In the first case we simulate 247 fictious clusters, each with 2360 random oriented members galaxies and compute mean value of analyzed statistics (i.e. in the discussed case χ^2 value). We give 1000 simulations and on this base we obtain: Cumulative Distribution Function (CDF) and Probability Density Function (PDF). Expected value of analyzed statistics and their variance is computed as well. Comparing them with theoretical prediction we are able to check our theoretical assumptions, correctness of the program and test quality of used random generator.

However, please note that the number of galaxies in our real clusters is small in some cases, and the χ^2 test will not necessarily work well (e.g. the χ^2 test requires the expected number of data per bin to equal at least 7; see, however, Snedecor & Cochran (1967); Domański (1979).) For this reason we repeat this procedure but now we simulate 247 fictious cluster each with number of members galaxies the same as in real clusters. As a check, we repeat the derivations for different values of n , but no significant difference appear so it is not presented in present paper.

Now we can compute the mean value of analyzed statistic for real sample of analyzed 247 rich Abell clusters and compare it with theoretical predictions and numerical simulations. This procedure is also provided for other, presented below, analyzed statistics.

The first auto-correlation test quantifies the correlations between galaxy numbers in neighboring angle bins. The measure of the correlation is defined as

$$C = \sum_{k=1}^n \frac{(N_k - N_{0,k})(N_{k+1} - N_{0,k+1})}{[N_{0,k}N_{0,k+1}]^{1/2}} \quad (2)$$

where $N_{n+1} = N_1$. Hawley & Peebles (1975) noted that in the case of an isotropic distribution, we expect $C = 0$ with the standard deviation:

$$\sigma(C) = n^{1/2} \quad (3)$$

Below we show that this result is an approximation which is not valid in our case.

Hawley & Peebles (1975) result was obtained on the assumption that all N_k are independent from each other. One should note that $E(\Sigma X) = \Sigma E(X)$ and moreover, if two variables X and Y are independent than we have: $E(XY) = E(X)E(Y)$, $D^2(X+Y) = D^2(X) + D^2(Y)$ and $D^2(XY) = D^2(X)D^2(Y) + (E(X))^2D^2(Y) + D^2(X)(E(Y))^2$. So, if all N_k are independent than we obtain:

$$\begin{aligned} E(C) &= E\left(\sum_{k=1}^n \frac{(N_k - N_{0,k})(N_{k+1} - N_{0,k+1})}{[N_{0,k}N_{0,k+1}]^{1/2}}\right) = \sum_{k=1}^n E\left(\frac{(N_k - N_{0,k})(N_{k+1} - N_{0,k+1})}{[N_{0,k}N_{0,k+1}]^{1/2}}\right) = \\ &= \sum_{k=1}^n E\left(\frac{N_k - N_{0,k}}{N_{0,k}^{1/2}}\right) E\left(\frac{N_{k+1} - N_{0,k+1}}{N_{0,k+1}^{1/2}}\right) = 0 \end{aligned} \quad (4)$$

and

$$\begin{aligned} D^2(C) &= D^2\left(\sum_{k=1}^n \frac{(N_k - N_{0,k})(N_{k+1} - N_{0,k+1})}{[N_{0,k}N_{0,k+1}]^{1/2}}\right) = \sum_{k=1}^n D^2\left(\frac{(N_k - N_{0,k})(N_{k+1} - N_{0,k+1})}{[N_{0,k}N_{0,k+1}]^{1/2}}\right) = \\ &= \sum_{k=1}^n D^2\left(\frac{N_k - N_{0,k}}{N_{0,k}^{1/2}}\right) D^2\left(\frac{N_{k+1} - N_{0,k+1}}{N_{0,k+1}^{1/2}}\right) = \sum_{k=1}^n 1 = n \end{aligned} \quad (5)$$

One should note however that the distribution of N_k is in fact a polynomial distribution and then elements of the covariance matrix of particular N_k are given by

the formulae $c_{ij} = N p_i(\delta_{ij} - p_j)$. If two variables X and Y are not independent than $E(XY) = E(X)E(Y) + Cov(XY)$. It leads to a conclusion that now in our formulae for $E(C)$ is present an additional term connected with covariance between value of N_k and N_{k+1} . As a result $E(C) = -\sum_{k=1}^n N p_k p_{k+1}$. Because in our case all p_k (and as a result also $N_{k,0}$) are equal ($p_k = 1/n$) than $E(C) = -\sum_{k=1}^n p_k = -\sum_{k=1}^n 1/n = -1$. Moreover when we try to compute $D^2(C)$ than variance of C contain term which is variance of products of N_k and N_{k+1} which are not independent. So, correct value of $D^2(C)$ is different from n and is obtained from numerical simulations.

Differences between our result and Hawley & Peebles (1975) approximation is not significant in the case of individual clusters because the difference between results in expected value of C (0 or -1) is small with comparison to its standard deviation $\sigma(C) \approx \sqrt{n}$. However one should note that in our case this difference is important. Our sample has 247 clusters so standard deviation of \bar{C} , $\sigma(\bar{C}) \approx \sqrt{n/247} = 0.3818$ is significantly smaller than a difference in expected values (which is equal to 1).

If deviation from isotropy is a slowly varying function of the angle θ one can use the Fourier test (Hawley & Peebles 1975)

$$N_k = N_{0,k}(1 + \Delta_{11} \cos 2\theta_k + \Delta_{21} \sin 2\theta_k) \quad (6)$$

We obtain the following expression for the Δ_{i1} coefficients

$$\Delta_{11} = \frac{\sum_{k=1}^n (N_k - N_{0,k}) \cos 2\theta_k}{\sum_{k=1}^n N_{0,k} \cos^2 2\theta_k}, \quad (7)$$

$$\Delta_{21} = \frac{\sum_{k=1}^n (N_k - N_{0,k}) \sin 2\theta_k}{\sum_{k=1}^n N_{0,k} \sin^2 2\theta_k}. \quad (8)$$

These equations are originally introduced by Hawley & Peebles (1975). It was written in a simple explicit form in the case when all N_k was equal and $n=36$ (equation 25 Hawley & Peebles (1975)).

Standard deviation of $\sigma(\Delta_{11})$ and $\sigma(\Delta_{12})$ is given by expressions:

$$\sigma(\Delta_{11}) = \left(\sum_{k=1}^n N_{0,k} \cos^2 2\theta_k \right)^{-1/2} = \left(\frac{2}{nN_0} \right)^{1/2}, \quad (9)$$

$$\sigma(\Delta_{21}) = \left(\sum_{k=1}^n N_{0,k} \sin^2 2\theta_k \right)^{-1/2} = \left(\frac{2}{nN_0} \right)^{1/2}. \quad (10)$$

The probability that the amplitude

$$\Delta_1 = (\Delta_{11}^2 + \Delta_{21}^2)^{1/2} \quad (11)$$

is greater than a certain chosen value is given by the formula

$$P(> \Delta_1) = \exp \left(-\frac{n}{4} N_0 \Delta_1^2 \right) \quad (12)$$

with standard deviation of this amplitude

$$\sigma(\Delta_1) = \left(\frac{2}{nN_0} \right)^{1/2}. \quad (13)$$

The formula for standard deviation for $\sigma(\Delta_{11})$, $\sigma(\Delta_{12})$, and $\sigma(\Delta_1)$ (Δ in the original Hawley & Peebles (1975) notation) was also written in a simple explicit form in Hawley & Peebles (1975) (equation 26). This test was substantially improved by Godłowski (1994) for the case when higher Fourier mode is taken into account: ¹

$$N_k = N_{0,k}(1 + \Delta_{11} \cos 2\theta_k + \Delta_{21} \sin 2\theta_k + \Delta_{12} \cos 4\theta_k + \Delta_{22} \sin 4\theta_k + \dots). \quad (14)$$

¹However please note that there is a printed error in Godłowski (1994). Eq. 18 should have form $P(\Delta) = (1 + J/2) \exp(-J/2)$

In our case (all $N_{0,k}$ are equal) it leads to formulas for the Δ_{ij} coefficients (Godłowski et al. 2010):

$$\Delta_{1j} = \frac{\sum_{k=1}^n N_k \cos 2J\theta_k}{\sum_{k=1}^n N_0 \cos^2 2J\theta_k}, \quad (15)$$

and

$$\Delta_{2j} = \frac{\sum_{k=1}^n N_k \sin 2J\theta_k}{\sum_{k=1}^n N_0 \sin^2 2J\theta_k}, \quad (16)$$

with the standard deviation

$$\sigma(\Delta_{1j}) = \left(\sum_{k=1}^n N_0 \cos^2 2J\theta_k \right)^{-1/2} = \left(\frac{2}{nN_0} \right)^{1/2}, \quad (17)$$

and

$$\sigma(\Delta_{2j}) = \left(\sum_{k=1}^n N_0 \sin^2 2J\theta_k \right)^{-1/2} = \left(\frac{2}{nN_0} \right)^{1/2}. \quad (18)$$

If we analyze Fourier modes separately, probability that the amplitude

$$\Delta_j = (\Delta_{1j}^2 + \Delta_{2j}^2)^{1/2} \quad (19)$$

is greater than a certain chosen value is given by the formula:

$$P(> \Delta_j) = \exp \left(-\frac{n}{4} N_0 \Delta_j^2 \right). \quad (20)$$

When we analyze first and second Fourier modes together the probability that the amplitude

$$\Delta = (\Delta_{11}^2 + \Delta_{21}^2 + \Delta_{12}^2 + \Delta_{22}^2)^{1/2} \quad (21)$$

is greater than a certain chosen value is given by the formulae

$$P(> \Delta) = \left(1 + \frac{n}{4} N_0 \Delta_j^2 \right) \exp \left(-\frac{n}{4} N_0 \Delta_j^2 \right). \quad (22)$$

The value of coefficient Δ_{11} gives us the direction of departure from isotropy. If, $\Delta_{11} < 0$ then the excess of the galaxies with position angles near 90 degrees is observed.

It means that in this case excess of galaxies with the position angel parallel to equatorial plane (case p) or parallel to Local Supercluster plane (case P) is observed, while for $\Delta_{11} > 0$ the excess of the galaxies with the position angle, respectively perpendicular to equatorial plane or Local Supercluster plane is observed.

In the paper Godłowski et al. (2010) the investigation of the linear regression given by $y = aN + b$ counted for various parameters was performed. In the case of position angles the linear regression between the values of statistics χ^2 , $\Delta_1/\sigma(\Delta_1)$, $\Delta/\sigma(\Delta)$ and the number of analyzed galaxies in each particular cluster was studied. It was found that non-randomness of galaxy orientation increased with numbers of objects in clusters. To test our null hypothesis H_0 that value of the analyzed statistics is as expected in the cases of random distribution of analyzed angles, we should now discuss properties of statistics $\Delta_1/\sigma(\Delta_1)$, $\Delta/\sigma(\Delta)$ as well as properties of the whole Fourier test in more details.

In one dimensional ($1D$) case the situation is very clear. Variables $\Delta_{11}/\sigma(\Delta_{11})$, $\Delta_{21}/\sigma(\Delta_{21})$ are normalized gaussian variables i.e. with expected value equal 0 i.e. $E(\Delta_{ij}/\sigma(\Delta_{ij}))=0$ and Variance equal 1 i.e. $D^2(\Delta_{ij}/\sigma(\Delta_{ij})) = 1$. Of course $\sigma^2(\overline{\Delta_{ij}/\sigma(\Delta_{ij})}) = 1/247 = 0.00405$ and $\sigma(\overline{\Delta_{ij}/\sigma(\Delta_{ij})}) = 0.06363$. However, in the case of $\Delta_1/\sigma(\Delta_1)$ and $\Delta/\sigma(\Delta)$ variables situation is much more complicated.

In our case, taking into account the equation 26 Hawley & Peebles (1975) (our equation 13) the equations 12 and 20 can be written (in analogy to $1D$ gaussian distribution) in the form:

$$P(> \Delta_j) = \exp\left(-\frac{1}{2} \frac{\Delta_j^2}{\sigma^2(\Delta_j)}\right). \quad (23)$$

while (having in mind that Δ is given by the equation 21) the equation 22 could be written as:

$$P(> \Delta) = \left(1 + \frac{1}{2} \frac{\Delta^2}{\sigma^2(\Delta)}\right) \exp\left(-\frac{1}{2} \frac{\Delta^2}{\sigma^2(\Delta)}\right). \quad (24)$$

Please note however that Δ_j is described by $2D$ Gaussian distribution while Δ is

described by $4D$ Gaussian distribution. In an explicit form the equation 12 could be written as:

$$P(> \Delta_1) = \exp \left(-\frac{1}{2} \left(\frac{\Delta_{11}^2}{\sigma^2(\Delta_{11})} + \frac{\Delta_{21}^2}{\sigma^2(\Delta_{21})} \right) \right). \quad (25)$$

So, the notation $\frac{\Delta_j^2}{\sigma^2(\Delta_j)}$ means only that elements of Δ^2 should be divided by elements of covariance matrix Δ_{ij} . Even more generally it could be written as:

$$P(> \Delta_1) = \exp \left(-\frac{1}{2} \sum_i \sum_j G_{ij} I_i I_j \right). \quad (26)$$

where I vector is

$$I = \begin{pmatrix} \Delta_{11} \\ \Delta_{21} \end{pmatrix} \quad (27)$$

and the matrix G is the inverse matrix to the covariance matrix of Δ_{ij} ($Cov = G^{-1}$). In the $4D$ case vector I has a form:

$$I = \begin{pmatrix} \Delta_{11} \\ \Delta_{21} \\ \Delta_{12} \\ \Delta_{22} \end{pmatrix} \quad (28)$$

while (using an auxiliary variable $J = \sum_i \sum_j G_{ij} I_i I_j$)

$$P(> \Delta) = (1 + J/2) \exp(-J/2) \quad (29)$$

Please note that the equation 26 is $2D$ equivalent of the equation 29 (and the equation 18 of Godłowski (1994) paper). Please note that the advantage of our notation is that it could be very easily extended on the situation that not all $\sigma(\Delta_{ij})$ are equal and/or non diagonal elements of covariance matrix are not disappear (i.e. not all Δ_{ij} are independent to each other).

One should note that the equation 13 (Hawley & Peebles (1975) equation 26) is obtained as a result of the theorem of propagation of errors. Because in our case

$\sigma(\Delta_{11}) = \sigma(\Delta_{21}) = (2/N)^{1/2}$ and $\Delta_1 = (\Delta_{11}^2 + \Delta_{21}^2)^{1/2}$ we obtain the following results:

$$\begin{aligned}\sigma^2(\Delta_1) &= \left(\frac{\partial \Delta}{\partial \Delta_{11}}\right)^2 \sigma^2(\Delta_{11}) + \left(\frac{\partial \Delta}{\partial \Delta_{21}}\right)^2 \sigma^2(\Delta_{21}) = \\ &= \left(\frac{2\Delta_{11}}{2\sqrt{\Delta_{11}^2 + \Delta_{21}^2}}\right)^2 \sigma^2(\Delta_{11}) + \left(\frac{2\Delta_{21}}{2\sqrt{\Delta_{11}^2 + \Delta_{21}^2}}\right)^2 \sigma^2(\Delta_{21}) = \\ &= \sigma^2(\Delta_{11}) = \sigma^2(\Delta_{21}) = 2/N\end{aligned}\tag{30}$$

i.e. our equation 13 (or Hawley & Peebles (1975) equation 26). We obtain analogical results in the case when we are taking into account both first and second Fourier modes $\sigma^2(\Delta) = \sigma^2(\Delta_{11}) = \sigma^2(\Delta_{21}) = \sigma^2(\Delta_{12}) = \sigma^2(\Delta_{22}) = 2/N$. However, please note that the theorem of propagation errors is obtained in the linear model. We argue below that a linear approximation is not good approximation in our case.

Please note that a value beeing the sum $\Delta_{ij}^2/\sigma^2(\Delta_{ij})$ is χ^2 distributed (Brandt 1997). As a result the value

$$\Delta_1^2/\sigma^2(\Delta_1) = \Delta_{11}^2/\sigma^2(\Delta_{11}) + \Delta_{21}^2/\sigma^2(\Delta_{21})\tag{31}$$

has χ^2 distribution with 2 degree of freedom while

$$\Delta^2/\sigma^2(\Delta) = \Delta_{11}^2/\sigma^2(\Delta_{11}) + \Delta_{21}^2/\sigma^2(\Delta_{21}) + \Delta_{12}^2/\sigma^2(\Delta_{12}) + \Delta_{22}^2/\sigma^2(\Delta_{22})\tag{32}$$

has χ^2 distribution with 4 degree of freedom. So, $E(\Delta_1^2/\sigma^2(\Delta_1)) = 2$, $E(\Delta^2/\sigma^2(\Delta)) = 4$, $D^2(\Delta_1^2/\sigma^2(\Delta_1)) = 4$ and $D^2(\Delta^2/\sigma^2(\Delta)) = 8$. Because we analyzed the $\Delta_1/\sigma(\Delta_1)$ and $\Delta/\sigma(\Delta)$ statistics, the expected value and the standard deviation of this statistics are of our interest. The theorem of propagation errors shows that

$$\sigma^2(x) = \left(\frac{\partial x}{\partial x^2}\right)^2 \sigma^2(x^2) = \left(\frac{1}{2\sqrt{x^2}}\right)^2 \sigma^2(x^2) = \frac{\sigma^2(x^2)}{4x^2}\tag{33}$$

In our case it leads to results that $\sigma(\Delta_1/\sigma(\Delta_1)) = 1/2$, and again $\sigma(\Delta/\sigma(\Delta)) = 1/2$. The expected value of $E(X) = \sqrt{E(X^2) - D^2(X)}$. In our case

$$E\left(\frac{\Delta_1}{\sigma(\Delta_1)}\right) = \sqrt{E\left(\frac{\Delta_1^2}{\sigma^2(\Delta_1)}\right) - D^2\left(\frac{\Delta_1}{\sigma(\Delta_1)}\right)} = \sqrt{2 - 0.5} = 1.2247\tag{34}$$

and

$$E\left(\frac{\Delta}{\sigma(\Delta)}\right) = \sqrt{E\left(\frac{\Delta^2}{\sigma^2(\Delta)}\right) - D^2\left(\frac{\Delta}{\sigma(\Delta)}\right)} = \sqrt{4 - 0.5} = 1.8708 \quad (35)$$

Because in our sample we have 247 clusters than $\sigma^2(\overline{\Delta_1/\sigma(\Delta_1)})$ and $\sigma^2(\overline{\Delta/\sigma(\Delta)})$ is equal $\frac{1/2}{247} = 0.002024$ while standard deviation of $\sigma(\overline{\Delta_1/\sigma(\Delta_1)})$ and $\sigma(\overline{\Delta/\sigma(\Delta)})$ is equal $\sqrt{\frac{1/2}{247}} = 0.04499$. These results are obtained from theorem of propagation errors, so again we assume linear approximation. We show in the next section that in presently analyzed case it works quite well, but correct values must be obtained from numerical simulations.

The isotropy of the resultant distributions of the angles θ can also be investigated using Kolmogorov- Smirnov test (K-S test). We assume that the theoretical, random distribution contains the same number of objects as the observed one. In such a case statistics λ

$$\lambda = \sqrt{n} D_n \quad (36)$$

is given by limit Kolmogorow distribution, where

$$D_n = \sup |F(x) - S(x)| \quad (37)$$

and $F(x)$ and $S(x)$ are theoretical and observational distributions of θ . Now we can compute mean value and standard deviation of analyzed statistic for real sample of analyzed 247 rich Abell clusters. Expected value of λ , its standard deviation as well as PDF and CDF of λ are obtained from numerical simulations.

4. Numerical Simulations

A well known problem with random number generators is that their quality is difficult to asses in any rigorous way. In fact many of the popular generators used till now failed to give correct results in multidimensional (sometimes even in two dimensional) simulations (Luescher 1994). We decided to test a few different number generators to be sure that

our result is correct. The first one is built in Fortran Lahey v3 generator. Moreover we used classical RAN1 generator with versions ($Ran1_{pt}$) of Numerical Recipes (Press et al. 1985) and more recent version ($Ran1_{nt}$) of Press & Teukolsky (1992), as well as GGUBS subroutine from ISML library, minimal standard generator discussed by Park & Miller (1988(@). We compared the result obtained using above generators with a result obtained with our basic generator which is a subtract-and-borrow random number generator originally proposed by Marsaglia & Zaman (1991) and later improved by Martin Luescher and called RANLUX (level 4) generator (Luescher 1994; James 1994).

The work of Martin Luescher provides the first operational definition of randomness in the sense required by Monte Carlo calculations and the Ranlux generator is the first one which produces random sequence in which no defect can be observed. The period of the generator is about 10^{171} . The RANLUX generator is based on a dynamical system which may be regarded as a multi-dimensional version of Arnold’s famous cat map. Similarly to the cat map, the system can be proved to be chaotic in a strong sense Luescher (1994, 2010).

Fundamentally different between traditional random number generators (TRNG) and RANLUX is that TRNGs must be tested because, apart from the testing, there is no reason to believe they are at all random. Experience shows that testing is necessary but not sufficient. RANLUX, on the other hand, has a good underlying theory, so the purpose of testing is only to make sure that the theory has been understood, applied and programmed correctly James (1994). Discussion of different types of Random Generator and advantages of RANLUX was discussed for example by Shchur & Butera (1998).

We performed 1000 simulations of 247 fictitious clusters, each with 2360 random oriented members galaxies. As a result we have the sample of $l = 1000$ values of particular statistics. For the present analysis we choose χ^2 , C and $\Delta_{11}/\sigma(\Delta_{11})$ statistics for which we have good

theoretical predictions only with an exception of the variance for C statistics which is obtained from the Peebles approximation. In this Table 1 we present average value of the analyzed statistics, its standard deviation and standard deviation in the sample. Moreover we present the standard deviation of the standard deviation estimator S which is equal $\sigma(S) = S/\sqrt{2(l-1)}$ (Brandt 1997).

One should note that RAN1 generator does not survive the tests. Version of Numerical Recipes (Press et al. 1985) gives wrong result of C statistics on more than 40σ level and wrong result of χ^2 on more than 30σ . Press & Teukolsky (1992) version is better but gives wrong result of $\Delta_{11}/\sigma(\Delta_{11})$ statistics on more than 10σ level, and more than 2σ deviation for χ^2 statistic. With built in Fortran Lahey v3 generator and GGUBS subroutine, a situation is much better, however some deviations from expected value (up to 2σ level) are observed. The RANLUX generator satisfies all three tests and we choose this generator as our base generator.

In our further analysis we choose six tests. We analyze χ^2 , $\Delta_1/\sigma(\Delta_1)$, $\Delta/\sigma(\Delta)$, C , λ and $\Delta_{11}/\sigma(\Delta_{11})$ statistics. In the Table 2 we present (as in the Table 1) average value of the analyzed statistics, its standard deviation, standard deviation in the sample as well as its standard deviation. Because of small number of galaxies in some cluster we repeat our analysis (Table 3) with 1000 simulations of 247 fictitious clusters, each cluster with number of members galaxies the same as in the real cluster. On the base of these simulations we built PDF and CDF presented in the Figures 1 and 2.

In our procedure we compute the mean values of the analyzed statistics. When the errors are normal (Gaussian), what is true at least in the case of $\Delta_{11}/\sigma(\Delta_{11})$ statistic, that parameters are estimated by maximum-likelihood method. They should have asymptotic normal (Gaussian) distribution. Now we check this suppositions using K-S test. For the test we choose the statistics $\bar{\chi}^2$ and $\overline{\Delta_{11}/\sigma(\Delta_{11})}$ because we have well theoretical predictions

about both expected values and variances of these statistics (one should note however that χ^2 statistics is χ^2 not normal distributed). As it was shown in the previous sections for $\bar{\chi}^2$ statistic $E(\bar{\chi}^2) = 35$ and $\sigma^2(\bar{\chi}^2) = 0.2834$ while for $\overline{\Delta_{11}/\sigma(\Delta_{11})}$ statistic $E(\overline{\Delta_{11}/\sigma(\Delta_{11})}) = 0$ and $\sigma^2(\overline{\Delta_{11}/\sigma(\Delta_{11})}) = 0.00405$. In order to reject the H_0 hypothesis, that distribution is Gaussian with expected value and variance as above, the value of observed statistics λ should be greater than λ_{cr} . At the significance level $\alpha = 0.05$ the value $\lambda_{cr} = 1.358$.

In the case when we performed 1000 simulations of 247 fictious clusters, each with 2360 random oriented members galaxies (Table 2) we obtained the values of statistic λ equal 0.5443 in the case of $\bar{\chi}^2$ statistic and 0.7963 in the case of $\overline{\Delta_{11}/\sigma(\Delta_{11})}$ statistic. When we repeated our analysis with 1000 simulations of 247 fictious clusters, each cluster with number of members galaxies the same as in the real cluster (Table 3) and Figures 1 and 2), we obtained values $\lambda = 0.9591$ and $\lambda = 0.7229$ respectively. All these values of λ are significantly less than $\lambda_{cr} = 1.358$. So we can not exclude our H_0 hypothesis. One should note however, that values of $\bar{\chi}^2$ and $\overline{\Delta_{11}/\sigma(\Delta_{11})}$ statistics obtained from numerical simulations ((Tables 2 and 3) are a little bit different from theoretical one.

Lilliefors (1967) showed that the standard tables used for the Kolmogorov-Smirnov test are valid when testing whether a set of observations are from a completely specified continuous distribution. When we check if the distribution is normal but one or more parameters are estimated from the sample then the Kolmogorov-Smirnov test no longer applies. At least it is not allowed to use the commonly tabulated critical points. It is suggested by Massey (1951) that if the test is used in this case, the results will be conservative in the sense that the probability of a type I error will be smaller than as given by tables of the Kolmogorov-Smirnov statistic. Lilliefors (1967) showed that the results of this procedure will indeed be extremely conservative and computed a new table for critical value of $D = \max|F^*(x) - S_N(x)|$ statistic, where $S_N(x)$ is the observational cumulative

distribution function and $F^*(x)$ is the cumulative normal distribution function with mean value and variance estimated from the sample. His critical values are about 30% less than those obtained by Massey (1951) for classical Kolmogorov-Smirnov test.

Above modification of the Kolmogorow test is usually known as Kolmogorow - Lilliefors test. For this test the critical value of D_{cr} , at the significance level $\alpha = 0.05$ for $n = 1000$ is equal 0.028. For analyzed statistics we obtained following values of D . For the sample of 247 fictious clusters, each with 2360 random oriented members galaxies (Table 2) we obtained $D = 0.014$ in the case of $\bar{\chi}^2$ statistic and $D = 0.018$ in the case of $\overline{\Delta_{11}/\sigma(\Delta_{11})}$ statistic. For sample of 247 fictious clusters, each cluster with number of members galaxies the same as in the real cluster (Table 3) we obtained $D = 0.018$ in the case of $\bar{\chi}^2$ statistic and $D = 0.0135$ in the case of $\overline{\Delta_{11}/\sigma(\Delta_{11})}$ statistic. Again all these values of λ are significantly less than critical value and again we can not exclude our H_0 hypothesis.

As a result we can conclude that analyzed statistics can be well described by the normal distribution. In particular it means that fluctuations observed in the Figure 2 in PDF of $\bar{\chi}^2$ statistics are not in conflict with our prediction that the statistics is normally distributed with parameters as obtained from theoretical predictions.

Independently we analyze situation (Table 4) when only galaxies brighter than $m_3 + 3$ are taken into account. Comparison of the Tables 2, 3 and 4 shows that there are some but not big differences between particular cases. When we compare average values of the statistics in the Tables 2 and 3 we found that in all cases differences between them are less than their 3 standard deviations, however for $\Delta_1/\sigma(\Delta_1)$, C and λ these differences are on the $2\sigma(\bar{x})$ level. It confirmed our suggestion that PDF and CDF used in further analysis should be built on the base of 1000 simulations of 247 fictious clusters, each cluster with number of members galaxies the same as in the real cluster. In the case of Tables 3 and 4 differences between average values are on $2\sigma(\bar{x})$ level only in the case of $\Delta/\sigma(\Delta)$ test. So it

is not necessary to build special PDF and CDF for this case.

5. Results

We analyzed the distribution of the position angles in the sample (A) of 247 rich Abell clusters both in Equatorial and Supergalactic coordinate system. Moreover we analyzed restricted sample (B) in which only galaxies brighter than $m_3 + 3$ are taken into account. The results are presented in the Table 5.

Our null hypothesis H_0 is that mean value of the analyzed statistics is as expected in the cases of a random distribution of the position angles, against H_1 hypothesis that analyzed values are different than in the case of random distribution. For the χ^2 test the result is significant on 3σ level, for autocorrelation test it is significant on 4σ level, while for Fourier test ($\Delta_1/\sigma(\Delta_1)$ and $\Delta/\sigma(\Delta)$ statistics) and Kolmogorow test (λ statistics) the results are significant on more than 5σ level. In all cases there are no significant differences when we analyzed distribution of Equatorial position angles p and Supergalactic position angles P . One can see from PDF and CDF presented in the Figures 1 and 2 that probability that such results are coming from random distributions is (in all cases) less than 0.1%.

$\Delta_{11}/\sigma(\Delta_{11})$ test does not show any difference from predictions of our null hypothesis H_0 which means that the average value of the analyzed statistic is as expected in the cases of a random distribution. This result, together with the fact that we show no significant differences with analysis in Equatorial and Supergalactic coordinate systems, shows that observed alignment is not connected with equatorial plane (as expected) nor with Supergalactic plane. Interpretation of this conclusion in the context of evolution of galaxies in the cluster needs detailed future investigation.

In our opinion it is because of the influence of environmental effects to the origin of

galaxy angular momenta. Godłowski et al. (2011) studied the galaxy alignment in the sample of very rich Abell clusters located in and outside superclusters. Even though that orientations of galaxies in analyzed clusters are not random, both in the case when we analyzed whole sample of the clusters and only clusters belonging to the superclusters, the statistically significant difference among investigated samples was found. In contrast to whole sample of cluster, where alignment increases with the cluster richness Godłowski et al. (2010) the cluster belonging to the superclusters does not show this effect. Moreover, the alignment decreases with the supercluster richness. The observed trend, dependence of galaxy alignment on both cluster location and supercluster richness clearly supports the influence of environmental effects to the origin of galaxy angular momenta.

Another important possibility is the influence of the large scale orientation of galaxy clusters (Hopkins et al. 2005; Wang et al. 2009; Godłowski & Flin 2010; Paz et al. 2011; Vera-Ciro et al. 2011; Hao et al. 2011; Schaefer & Merkel 2011; Smargon et al. 2011; Blazek, McQuinn & Seljak 2011; Song & Lee 2011; Noh & Cohn 2011) analyzed both theoretically and observationally. Godłowski & Flin (2010) studied the orientation of galaxy groups in the Local Supercluster (LSC). It is strongly correlated with the distribution of neighbouring groups in the scale till about 20 Mpc. Paz et al. (2011) found a strong alignment between the projected major axis of group shapes and the surrounding galaxy distribution up to scales of $30 Mpc/h$. Smargon et al. (2011) search for two types of cluster alignments using pairs of clusters: the alignment between the projected major axes of the clusters founding weak effect up to $20 Mpc/h$, and the alignment between one cluster major axis and the line connecting it to the other cluster in the pair founding strong alignment on scales up to $100 Mpc/h$.

The change of alignment with the surrounding neighbourhood was observed also in alignment study in void vicinity (Varela et al. 2011) being continuation of earlier study of

galaxy orientation in regions surrounding bubble-like voids (Trujillo et al. 2006). Another interesting result was found by Jones et al. (2010) who reported that the spins of spiral galaxies located within cosmic web filaments tend to be aligned along the larger axis of the filament, which is interpreted by the authors as "fossil" evidence indicating that the action of large scale tidal torques effected the alignments of galaxies located in cosmic filaments.

For the sample B the results are weaker but still significant. As above, $\Delta_{11}/\sigma(\Delta_{11})$ does not show any difference from predictions of our null hypothesis H_0 . For χ^2 test the result is significant at 2σ level while for remaining four tests results are significant on more than 3σ level.

Our analysis leads to the conclusion that we observed significant alignment of galaxies in our sample of rich Abell clusters. One should note that the most powerful test is the Fourier test. It is not a surprise because during previous analysis of galactic alignment starting from Hawley & Peebles (1975) Fourier test was the most sensitive one. Nearly the same significance level as Fourier test shows the Kolmogorow test. One should note, that in the contrast to analysis of individual structures where the autocorrelation test usually does not lead to a significance conclusion (see for example (Godłowski 1993, 1994; Godłowski et al. 2005)), during our analysis the autocorrelation test is more powerful than the χ^2 test.

6. Conclusions

We investigated statistical tests originally proposed by Hawley & Peebles (1975) for analysis of the galactic orientations. Basing on analyzed tests, the method of analysis of the alignment of galaxies in clusters was proposed. We analyzed the alignment of galaxies belonging to 247 Abell clusters containing at least 100 members. The distributions of the

position angles for galaxies in each cluster were analyzed using statistical tests: χ^2 test, Fourier tests, Autocorrelation test and Kolmogorow test. The mean value of the analyzed statistics was compared with theoretical predictions as well as with results obtained from numerical simulations.

The statistical tests originally proposed by Hawley & Peebles (1975) for analysis of the galactic orientations were the χ test, the Fourier test and the autocorrelation test. We analyzed the autocorrelation test in more detail and some improvements were proposed. It was shown that original Hawley & Peebles (1975) result is an approximation which is not fully valid in our case. We pointed out that the distribution of the number of galaxies with orientations within the k -th angular bin N_k is in fact a polynomial distribution and then in particular N_K are not independent to each other. In the result the expected value of C statistics is equal -1 instead 0 as in original Hawley & Peebles (1975) paper. This difference is not significant in the case of individual clusters because it is small with comparison to its standard deviation $\sigma(C) \approx \sqrt{n}$. However in our case when we analyze 247 clusters and compute the average value of the statistics this difference begin to be important. This is because variance of average values $\sigma(\bar{C}) \approx \sqrt{n/247} = 0.3818$ starts to be significantly smaller than a difference between our and approximated by Peebles expected values of C . Separately we analyzed in detail the Fourier test. We analyzed the properties of whole Fourier test as well as the $\Delta_1/\sigma(\Delta_1)$, $\Delta/\sigma(\Delta)$ and $\Delta_{11}/\sigma(\Delta_{11})$ statistics. We compute the expected value and the variance of these statistics. The results of our theoretical investigations were compared with the numerical simulations.

Our analysis of the distributions of the position angles of galaxies in rich Abell clusters shows that the orientation of galaxies in analyzed cluster is not random i.e. we found an existence of alignment of galaxies in the rich Abell galaxy clusters. Five statistical test show that distribution of the position angles is not random at least on 3σ level. In all cases there

was no significant difference when we analyzed distribution of Equatorial position angles p and Supergalactic position angles P . Moreover $\Delta_{11}/\sigma(\Delta_{11})$ statistics do not show any significant deviation from randomness. These two facts suggest that observed alignment is not connected with equatorial plane (as expected) nor with Supergalactic plane.

Our previous analysis (Godłowski & Flin 2010; Godłowski 2011; Flin et al. 2011) shows the dependency on the alignment of galaxies in clusters and richness of the cluster which leads to the conclusion that the angular momentum of the cluster increases with the mass of the structure. With such a dependency it is natural to expect that in rich clusters significant alignment should be present. In the present paper we confirmed this prediction.

Usually a dependence between the angular momentum and the mass of the structure is presented as empirical relation $J \sim M^{5/3}$ (Wesson 1979, 1983; Carrasco 1982; Brosche 1986). In our opinion the observed relation between the richness of the galaxy cluster and the alignment is due to tidal torque, as suggested by Heavens & Peacock (1988) and Catelan & Theuns (1996). Moreover, the analysis of the linear tidal torque theory is pointing in the same direction (Noh & Lee 2006a,b). They noticed the connection of the alignment with the considered scale of the structure. However one should note that our result is also compatible with the prediction of the Li model (Li 1998; Godłowski et al. 2003, 2005) in which galaxies form in the rotating universe.

In our further paper we would like to extend our consideration to analysis of the distribution of two angles δ_D (the angle between the normal to the galaxy plane and the main plane of the coordinate system), and η (the angle between the projection of this normal onto the main plane and the direction towards the zero initial meridian) describing the spatial orientation of the galaxy plane. Moreover we would like to investigate if effect found in the present paper depends on the cluster BM type.

Acknowledgments

This research has made use of the NASA/IPAC Extragalactic Database (NED) which is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. Author thanks Elena Panko and Piotr Flin for permission to use unpublished data from their catalog and Stanisław Jadach for drawing my attention to RANLUX generator and for helpful remarks and discussion on this problem. The author thanks anonymous referee for detailed remarks which helped to improve the original manuscript.

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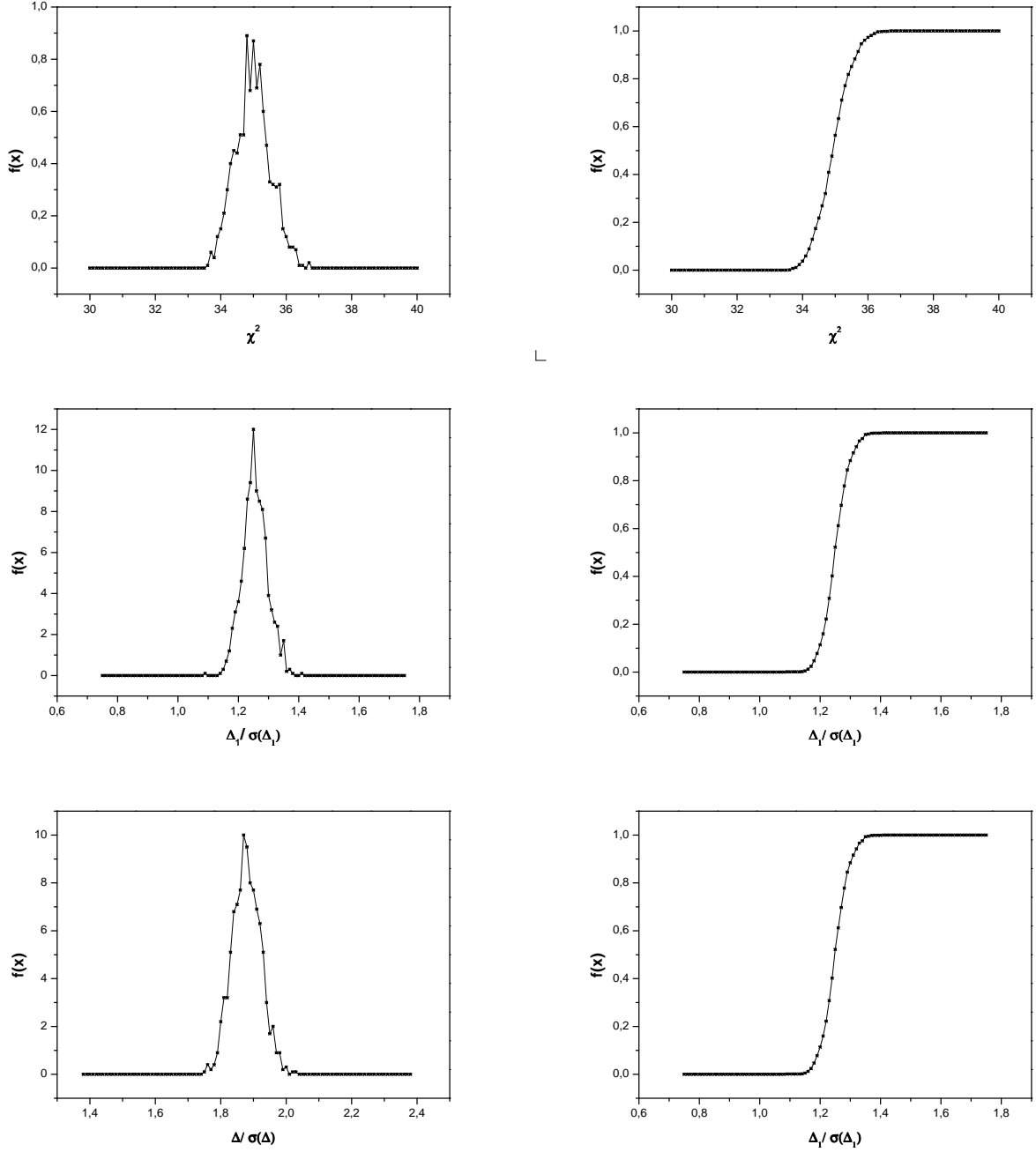


Fig. 1.— The Probability Density Function (PDF) (left panel) and Cumulative Distribution Function (CDF) (right panel) for analyzed statistics. The figure was obtained from 1000 simulations of sample of 247 cluster each with number of members galaxies the same as in the real cluster. From up to down we present statistics: χ^2 , $\Delta_1/\sigma(\Delta_1)$, $\Delta/\sigma(\Delta)$.

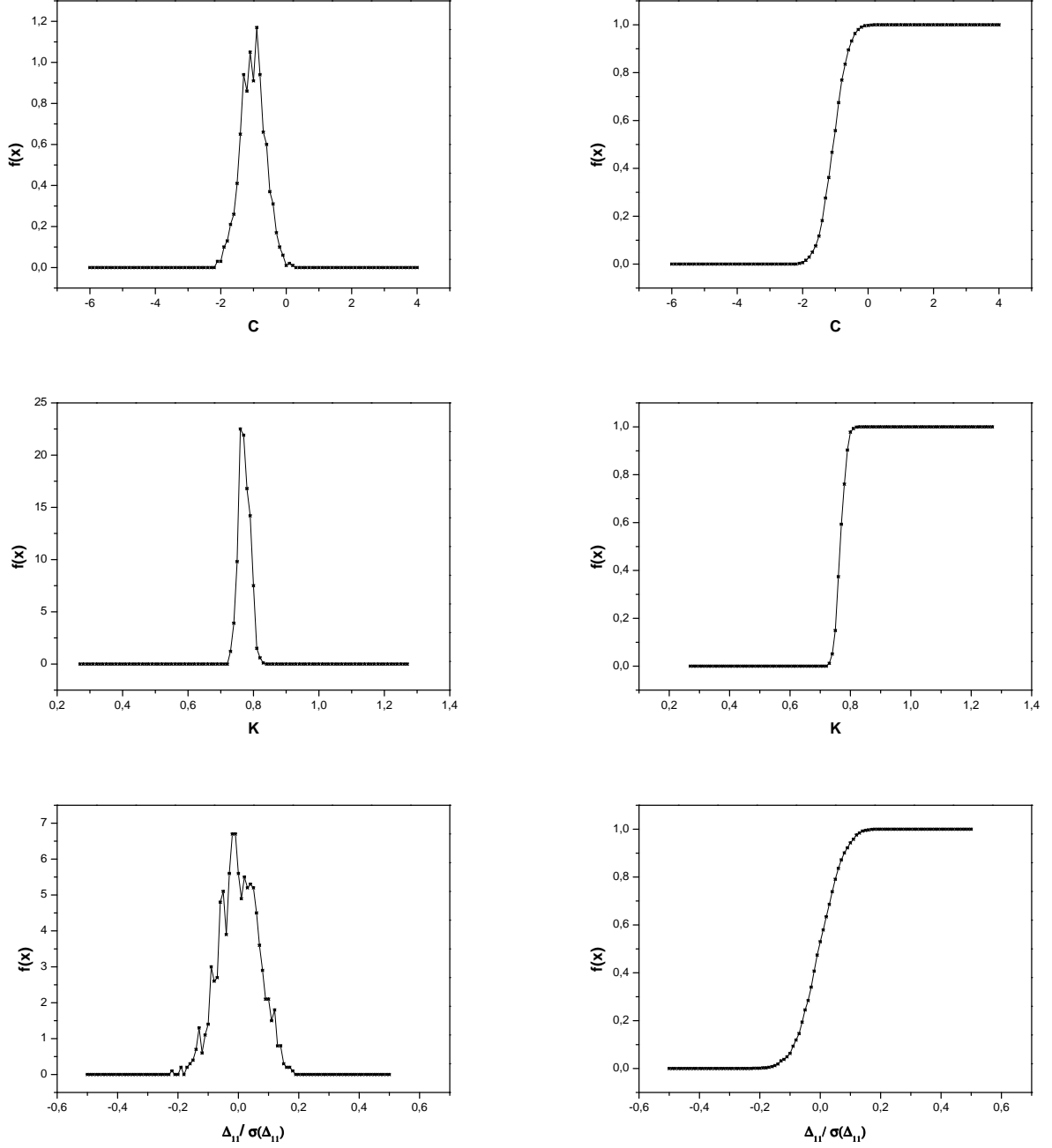


Fig. 2.— The Probability Density Function (PDF) (left panel) and Cumulative Distribution Function (CDF) (right panel) for analyzed statistics. The figure was obtained from 1000 simulations of sample of 247 cluster each with number of members galaxies the same as in the real cluster. From up to down we present statistics: C , λ , $\Delta_{11}/\sigma(\Delta_{11})$.

Table 1: The comparison of different numerical generators

Generator	Test	\bar{x}	$\sigma(x)$	$\sigma(\bar{x})$	$\sigma(\sigma(x))$
<i>Lahey</i>	χ^2	34.9741	0.5271	0.0166	0.0117
	C	−0.9981	0.3743	0.0118	0.0083
	$\Delta_{11}/\sigma(\Delta_{11})$	−0.0040	0.0616	0.0019	0.0014
<i>Ran1_{pt}</i>	χ^2	34.4993	0.5157	0.0163	0.0115
	C	−0.5145	0.3569	0.0113	0.0080
	$\Delta_{11}/\sigma(\Delta_{11})$	0.0018	0.0613	0.0019	0.0014
<i>Ran1_{nr}</i>	χ^2	34.9604	0.5320	0.0168	0.0119
	C	−1.0018	0.3836	0.0121	0.0086
	$\Delta_{11}/\sigma(\Delta_{11})$	0.0282	0.0632	0.0020	0.0014
<i>GGUBS</i>	χ^2	34.9874	0.5505	0.0174	0.0123
	C	−1.0110	0.3765	0.0119	0.0084
	$\Delta_{11}/\sigma(\Delta_{11})$	0.0021	0.0637	0.0020	0.0014
<i>RANLUX</i>	χ^2	34.9978	0.5442	0.0172	0.0121
	C	−0.9917	0.3899	0.0123	0.0087
	$\Delta_{11}/\sigma(\Delta_{11})$	−0.0010	0.0643	0.0020	0.0014

Table 2: The result of numerical simulation - sample of 247 cluster each with 2360 galaxies

Test	\bar{x}	$\sigma(x)$	$\sigma(\bar{x})$	$\sigma(\sigma(x))$
χ^2	34.9978	0.5442	0.0172	0.0121
$\Delta_1/\sigma(\Delta_1)$	1.2524	0.0424	0.0013	0.0009
$\Delta/\sigma(\Delta)$	1.8794	0.0460	0.0014	0.0010
C	−0.9917	0.3899	0.0123	0.0087
λ	0.7708	0.0166	0.0005	0.0003
$\Delta_{11}/\sigma(\Delta_{11})$	−0.0010	0.0643	0.0020	0.0014

Table 3: The result of numerical simulation - sample of 247 cluster each with number of members galaxies the same as in the real cluster.

Test	\bar{x}	$\sigma(x)$	$\sigma(\bar{x})$	$\sigma(\sigma(x))$
χ^2	34.9798	0.5364	0.0170	0.0120
$\Delta_1/\sigma(\Delta_1)$	1.2550	0.0419	0.0013	0.0009
$\Delta/\sigma(\Delta)$	1.8788	0.0436	0.0014	0.0010
C	-1.0195	0.3749	0.0119	0.0084
λ	0.7720	0.0168	0.0005	0.0004
$\Delta_{11}/\sigma(\Delta_{11})$	0.0014	0.0645	0.0020	0.0014

Table 4: The result of numerical simulation - sample of 247 cluster each with number of members galaxies the same as in the real cluster but only galaxies brighter than $m_3 + 3$ are taken into account.

Test	\bar{x}	$\sigma(x)$	$\sigma(\bar{x})$	$\sigma(\sigma(x))$
χ^2	34.9902	0.5309	0.0168	0.0119
$\Delta_1/\sigma(\Delta_1)$	1.2564	0.0418	0.0013	0.0009
$\Delta/\sigma(\Delta)$	1.8820	0.0434	0.0014	0.0010
C	−1.0007	0.3716	0.0118	0.0083
λ	0.7724	0.0163	0.0005	0.0004
$\Delta_{11}/\sigma(\Delta_{11})$	−0.0006	0.0636	0.0020	0.0014

Table 5: The value of analyzed statistics real sample of 247 Abell clusters.

Sample	Test	Equatorial coordinates		Supergalactic coordinates	
		\bar{x}	$\sigma(\bar{x})$	\bar{x}	$\sigma(\bar{x})$
A	χ^2	36.8591	0.5924	36.7899	0.6315
	$\Delta_1/\sigma(\Delta_1)$	1.7046	0.0622	1.7021	0.0626
	$\Delta/\sigma(\Delta)$	2.2663	0.0594	2.2746	0.0591
	C	1.1940	0.4530	1.1220	0.4237
	λ	0.9177	0.0240	0.9138	0.0220
	$\Delta_{11}/\sigma(\Delta_{11})$	−0.0005	0.0855	0.0940	0.0924
B	χ^2	36.4000	0.6072	36.2919	0.6124
	$\Delta_1/\sigma(\Delta_1)$	1.6283	0.0577	1.6316	0.0578
	$\Delta/\sigma(\Delta)$	2.2055	0.0565	2.2199	0.0554
	C	0.8843	0.4355	0.7863	0.4212
	λ	0.8928	0.0224	0.8934	0.0210
	$\Delta_{11}/\sigma(\Delta_{11})$	0.0023	0.0826	0.0810	0.0866